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# TECHNICAL NOTE

D-1262

STATISTICAL STUDY OF EFFECTS OF INSERTION ERRORS ON  
ASCENT TRAJECTORIES FOR LUNAR MISSIONS

By Harold A. Hamer, Carl R. Huss, and John P. Mayer

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION  
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## SUMMARY

The effects of insertion errors on the success of lunar missions were determined from a statistical analysis. With the use of the Monte Carlo technique, random uncorrelated errors were considered to exist both simultaneously and singly in the four insertion conditions (velocity, altitude, flight-path angle, and lead angle) required for establishing two-dimensional lunar ballistic trajectories. The trajectories were calculated by the use of the restricted two-body orbit equations and the sphere-of-influence concept. The insertion errors were assumed to be normally distributed; three specified values for the standard deviation of each quantity were covered in the analysis. The effects of these errors were analyzed for trajectories designed for lunar impact as well as for those intended to pass the moon at certain distances. Accuracy considerations determined from the present analysis are generally applicable to any lunar mission performed with ballistic trajectories, regardless of the method used for calculating the trajectories.

For trajectories designed for lunar impact, results are shown as probabilities of missing the moon. Data are also included on impact-angle probabilities and impact-longitude dispersion. For trajectories designed to miss the moon, results are shown as probabilities of hitting the moon and as dispersion in perilune distance. The results of the analysis show that, except for trajectories designed for relatively low insertion velocities, accuracy requirements in lead angle and insertion flight-path angle are the most important for a successful mission. The effects of the nominal values of insertion radius and insertion flight-path angle on the success of a mission, as well as on the impact-angle probabilities and dispersion values, are negligible. For trajectories designed for lunar impact, the nominal value of insertion velocity has an appreciable effect on the mission success and on the impact-longitude dispersion, but only a small effect on the impact-angle probabilities. For trajectories designed to miss the moon, the success of the mission generally is not affected by the nominal value of insertion velocity.

## INTRODUCTION

For any space mission, because of errors in the many controlling parameters, the insertion conditions achieved at burnout will differ from the exact requirements for the nominal (planned) trajectory. While it is generally agreed that midcourse guidance is necessary for manned or unmanned lunar missions, studies of ballistic (unguided) trajectories have been employed for determining accuracy requirements of lunar vehicles. Attention has been given to errors in perilune distance resulting from discrete errors in the insertion conditions. (For example, see refs. 1 to 6.) For more comprehensive information on accuracy requirements, statistical studies of errors in perilune distance resulting from random insertion errors are essential. Several approaches to this problem are given in references 7 and 8.

This paper presents the results of a Monte Carlo analysis of insertion errors associated with lunar ballistic trajectories. The trajectory calculations used in this analysis are based on equations given in reference 9. The results are based on several ranges of inaccuracies which might be present in the guidance-system and rocket-engine components of the lunar vehicle. For the analysis, the random errors are assumed to be uncorrelated and to exist simultaneously in all the insertion conditions. In addition, the random errors in the insertion conditions are introduced singly to show the relative importance of each of these parameters. In addition to showing the probable effects of random errors of various root-mean-square magnitudes on the ascent trajectories for lunar missions, the results indicate how the selection of insertion-condition values for a given mission can be expected to affect the success of the mission.

Two general types of trajectories are studied: those which, without insertion errors, would intersect the center of the moon and those which would pass the moon at a given distance. A wide range of values of insertion conditions is analyzed in each case.

## SYMBOLS

The English system of units is used in this study. In case conversion to metric units is desired, the following relationships apply: 1 foot = 0.3048 meter, and 1 statute mile = 5,280 feet = 1,609.344 meters.

$\Delta L_I$  lunar impact-longitude dispersion about nominal impact longitude (measured in plane of vehicle motion), deg

N      magnitude of standard deviation of insertion error

$r_e$	insertion radius from center of earth, statute miles
$r_{m,p}$	perilune distance (closest approach of vehicle to center of moon), statute miles
$\Delta r_{m,p}$	perilune-distance dispersion about nominal perilune distance (measured in plane of vehicle motion), statute miles
$V_e$	vehicle inertial velocity at insertion, ft/sec
$V_{e,min}$	minimum vehicle insertion velocity for lunar impact for given value of $r_e$ , ft/sec
$\Delta V_e$	incremental insertion velocity, $V_e - V_{e,min}$ , ft/sec
$\beta$	lunar impact angle; angle between velocity vector and local horizontal to moon, deg
$\gamma_e$	insertion flight-path angle; angle between velocity vector and local horizontal to earth, deg
$\sigma$	standard deviation
$\tau$	lunar lead angle; angle between geocentric radius vector and line joining centers of earth and moon at insertion, deg

The subscript H signifies values of insertion conditions for impact trajectories designed to intersect the center of the moon (dead-center hit).

## METHOD

### Trajectory Calculations

Details of the method used for the trajectory calculations are described completely in reference 9. For this method, two-dimensional (coplanar) lunar ballistic trajectories are calculated by using the restricted two-body orbit equations. A sphere of influence is assumed to exist about the moon so that the attractions of the earth and moon on the vehicle can be treated separately by use of the two-body equations. The earth and moon are taken as point masses and the moon is considered to revolve about the center of the earth at its average distance. The results presented in this report are considered to be generally applicable to any statistical study of lunar-ballistic-trajectory accuracy requirements. Data given in reference 9 indicate

that results obtained by the two-body method are sufficiently accurate to permit use of this simplified method for a statistical study of the nature reported herein. The simplified trajectory calculations greatly reduced the machine time ordinarily required for the large number of solutions needed in the statistical analysis.

Insertion conditions.- The values of insertion conditions required for a wide variety of lunar ballistic trajectories are presented in reference 9. The insertion conditions at burnout which completely describe the two-dimensional trajectories are (1) lunar lead angle or firing time, (2) velocity, (3) radius vector or altitude, and (4) flight-path angle.

In order to provide a general background of lunar-ballistic-trajectory requirements, examples of some typical combinations of insertion-condition values required for certain perilune distances are shown in figures 1 and 2. The curves in figure 1 show variations of insertion velocity and lead angle for given perilune distances. The data in this figure apply to one value of insertion radius and one value of insertion flight-path angle. The data in figure 2 show velocity and lead-angle requirements for trajectories designed to intersect the center of the moon for various insertion radii and insertion flight-path angles.

Nominal trajectories.- A "nominal" trajectory is defined herein as a trajectory that accomplishes a given mission exactly as planned; that is, there are no errors in the insertion conditions. Two types of trajectories are considered: those which intersect the center of the moon and those which miss the moon by a specified distance. Values of the insertion conditions for the nominal trajectories used in this analysis were selected from calculations made for the study reported in reference 9. Figures 1 and 2 are examples of these calculations and illustrate the wide selection of nominal trajectories that are available for lunar missions.

Lunar ballistic trajectories can be separated into two broad categories: ascending and descending. (See fig. 1.) Ascending trajectories hit the moon or reach perilune (closest approach to the moon) while the vehicle is still moving away from the earth; the opposite is true for descending trajectories. In the present analysis only ascending trajectories are studied inasmuch as these are considered the more practical because of the comparatively shorter trip times to the moon. (See ref. 9 for a complete description of lunar trajectory characteristics.) Also, for the nominal trajectories designed to miss the moon, only those are studied in which the vehicle motion at perilune is clockwise with respect to the moon (see fig. 1); that is, trajectories that intersect the lunar orbit ahead of the moon. In contrast to the trajectories with counterclockwise vehicle motion, these clockwise trajectories are circumlunar (revolve around the far side of the moon), in most cases return

to relatively short distances from the earth, and represent comparatively short total trip times from insertion to return. It should be stated that these trajectories revolve around the moon and return to earth only in the range of insertion velocities below the escape value.

### Error Analysis

The curves in figure 3 illustrate the effects of errors in each of the insertion conditions on perilune distance. The data represent only one set of nominal insertion values; however, there are similar trends in the perilune-distance errors for other sets of insertion conditions. It is seen in the figure that the curves are both nonlinear and unsymmetrical about the nominal values. As a result of these two characteristics, methods of statistical-error analysis requiring use of the partial derivatives of these quantities (linear perturbation theory) could not be accurately applied in the present study. In order to determine accurately the errors in perilune distance associated with various insertion errors, a random procedure was required for determining errors in each insertion condition, with a new trajectory to the moon being calculated for each set of insertion errors.

The Monte Carlo method was used to determine the random (uncorrelated) errors in the insertion conditions. The errors in each of the insertion conditions were assumed to be normally distributed (Gaussian distribution). In applying the Monte Carlo method, random numbers were used to determine values of insertion errors from these distributions. These values, in turn, were added to the insertion-condition values for the nominal trajectories.

In the analysis, three specific values of the standard deviation of each insertion-condition error were used. In one case, the value specified was that considered to be typical of lunar-vehicle systems. This value is designated by the letter  $N$ . For the other two cases,  $N$  was multiplied by 4 and by  $1/2$  to represent systems with lower and higher degrees of accuracy, respectively. The values of the standard deviations used in each case are given in the following table:

Insertion conditions	Standard deviation, $\sigma$		
	$N$	$4N$	$N/2$
$r_e$ , statute miles	0.25	1	0.125
$V_e$ , ft/sec	5	20	2.5
$\gamma_e$ , deg	.1	.4	.05
$\alpha_\tau$ , deg	.25	1	.125

<sup>a</sup> 0.25° in lunar lead angle represents approximately 1 minute in launch time.

For each nominal trajectory in this analysis, 1,000 sets of insertion errors were used to obtain 1,000 samples (trajectories). In addition, for several cases the calculations were made for 5,000 sets of errors. Examples of these calculations are shown in figure 4 as probability distributions of perilune distance. In figure 4(a) the nominal trajectory will intersect the center of the moon. In figure 4(b) the nominal trajectory will miss the center of the moon by 6,000 statute miles. The probability distributions are shown for 1,000 and 5,000 samples (trajectories) and the close comparison for these data indicates that 1,000 trajectory calculations are sufficient for determining a true representation of the perilune-distance probability values. The statistical results of the analysis are, for the most part, based on probability values selected from curves such as shown in figure 4.

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## RESULTS AND DISCUSSION

The main results from the statistical study are presented in figures 5 to 13. Figures 5 to 11 pertain to accuracies of ballistic trajectories designed for lunar impact; figures 12 and 13 pertain to trajectories designed to miss the moon by certain distances. With the exception of the data in figure 9, random errors were considered to exist simultaneously in all four insertion conditions. For figure 9, random errors in the insertion conditions were considered separately in order to show the relative importance of each insertion condition in the success of a mission.

### Lunar Impact Missions

Probability of missing the moon.- Figure 5 presents data on probability of missing the moon for ballistic trajectories designed (no insertion errors) to intersect the center of the moon. Each data point is the probability of exceeding a perilune distance of 1,080 statute miles (radius of the moon) as determined from a probability distribution such as that shown in figure 4(a). The data in figure 5 are shown for three sets of standard-deviation values as given in the table in the preceding section. Probability values are shown for a large collection of nominal insertion conditions: one insertion flight-path angle, two insertion radii, and a wide range of insertion velocities.

In subsequent figures the data are normalized by subtracting from the insertion velocity  $V_e$  the minimum insertion velocity required for lunar impact  $V_{e,min}$ . This difference, defined as  $\Delta V_e$ , is determined from  $V_{e,min}$  values shown in figure 6. The  $V_{e,min}$  values (obtained from ref. 9) are primarily a function of insertion radius; the effects

of insertion flight-path angle up to the limit of the study can be considered negligible.

The results of figure 5 are shown in figure 7 in the normalized form with an additional dashed curve representing calculations made for an increase in the velocity error only. When based on  $\Delta V_e$ , the probability of missing the moon is seen to be independent of the nominal value of insertion radius. On the basis of normalized results in reference 9, the data in figure 7 and in subsequent figures are applicable to insertion radii up to at least 5,000 statute miles. For trajectories designed to pass through the center of the moon, it can be seen in figure 7 that the nominal insertion velocity has a noticeable effect on the success of a mission. (A successful mission is defined here as one in which the vehicle does not miss the moon.) The probability of success, which is one minus the probability value shown in the figure, is highest for insertion velocities between the minimum value for impact and the escape-velocity value. (The value of  $\Delta V_e$  corresponding to escape velocity differs slightly with insertion radius.) In this range of insertion velocities having the highest probability of success ( $\Delta V_e = 50$  to  $200$  ft/sec), it can be seen that decreasing the insertion errors from  $\sigma = N$  to  $\sigma = N/2$  does not materially change the probability of success.

For the solid curves in figure 7, the relative ratios between the standard deviations of each insertion-condition error are the same for the three different magnitudes of error; therefore, the minimum point of each curve occurs at the same value of incremental insertion velocity. Changing these ratios may shift this point of greatest success to a different value of insertion velocity. For example, an increase in the standard deviation of velocity error will shift this point to a higher value of  $\Delta V_e$ , as shown by the dashed curve.

The effect of the nominal value of insertion flight-path angle on the probability of success of an impact mission was investigated. The results are shown in figure 8 for flight-path angles from  $0^\circ$  to  $60^\circ$ . Included in the figure are probability values for two sets of standard deviations for the insertion errors and three values of the incremental insertion velocity. It is apparent from the figure that the probability of success of any ballistic impact mission is essentially unaffected by flight-path angle (at least up to  $\gamma_e = 60^\circ$ ).

The effects of errors in each insertion condition are shown in figure 9 as a function of insertion velocity. These results apply to any nominal value of insertion radius or insertion flight-path angle. For each curve shown, random errors are allowed to exist only in the corresponding insertion condition; the other three insertion conditions are at the exact values required for a dead-center hit. The largest

value for the standard deviation ( $\sigma = 4N$ ) was used for purposes of best illustrating the individual effects. As seen in the figure, the relative insertion accuracy requirements for a successful mission change with the nominal value of insertion velocity. Effects of insertion-radius error are the least important. Effect of insertion-velocity error is important for trajectories with low values of insertion velocity, whereas accuracy considerations are important in both lead angle and flight-path angle for trajectories designed for higher insertion velocities. The effects of errors in lead angle and flight-path angle are seen to be about equal for the particular ratio assumed between the standard deviations of these two quantities.

Dispersion in impact longitude.- Data on dispersion in the lunar-impact longitude are shown in figure 10. The data are shown as a function of insertion velocity for the three specified sets of standard deviations. The impact-longitude dispersions about the nominal impact longitude for ballistic trajectories designed for dead-center hits are shown for probabilities of 0.5 and 0.9. (As noted in fig. 7, some of the trajectories do not hit the moon.) For example, in figure 10, for  $\Delta V_e = 1,200$  ft/sec and  $\sigma = N$ , the probability is 0.5 that the impact longitude will be within  $29^\circ$  of the nominal value. The dispersions were determined from the probability distributions of lunar-impact longitude. It was found that for normally distributed insertion errors, the impact longitudes were approximately normally distributed about the nominal impact longitudes. The data given in figure 10 apply to all nominal values of insertion radius and insertion flight-path angle. Also shown in figure 10 is the maximum possible dispersion (for large insertion error) for ballistic impact (taken from ref. 9). In all cases, lunar longitude is measured in the plane of vehicle motion (earth-moon plane).

The results in figure 10 show that even very small insertion errors produce a relatively large dispersion in the impact longitude. Also, there is an effect of the nominal value of insertion velocity on impact-longitude dispersion; the largest amounts of dispersion can be expected for trajectories designed for insertion velocities near the minimum value required for impact. For the two cases with the smallest insertion errors ( $\sigma = N$  and  $N/2$ ), the least dispersion occurs at  $\Delta V_e = 150$  ft/sec. This value of  $\Delta V_e$  corresponds to values of insertion velocity which provide the lowest probability of missing the moon. (See fig. 7.) For trajectories designed for the higher insertion velocities ( $\Delta V_e > 400$  ft/sec), dispersion is essentially unaffected by the nominal insertion velocity. Also, at these insertion speeds, it is of interest to note that increasing the insertion errors from  $\sigma = N$  to  $\sigma = 4N$  does not significantly change the impact-longitude dispersion.

Impact angle.- Probability distributions of impact angle for lunar ballistic trajectories designed for a dead-center hit (Impact angle =  $90^\circ$ )

are presented in figure 11. The effects of insertion accuracy and nominal insertion velocity on the probability of impact angle being greater than a given value are shown in figures 11(a) and 11(b), respectively. The probability distributions apply to all nominal values of insertion radius and insertion flight-path angle, inasmuch as the effects of these two variables on the impact-angle probabilities were found to be negligible.

For purposes of illustration, the probability data in figure 11(a) are shown for nominal insertion velocities corresponding to  $\Delta V_e = 150$  ft/sec. Comparison of the curves for the three specified sets of standard deviations indicates a large effect of insertion accuracy on the impact angle. For example, of the trajectories which hit the moon, the impact angle will be greater than  $80^\circ$  for 50 percent of the time for highly accurate insertions ( $\sigma = N/2$ ) and only about 20 percent of the time for insertions of poor accuracy ( $\sigma = 4N$ ).

The data in figure 11(b) show a relatively small effect of nominal insertion velocity on the probability of achieving given ballistic impact angles. As indicated in the figure, there is no effect for values of  $\Delta V_e$  from 40 to 150 ft/sec. Insertion velocities in this range provide the best chance of achieving a high impact angle. As the insertion velocity is increased above  $\Delta V_e = 150$  ft/sec, the probability values decrease (approximately linearly) until a value of  $\Delta V_e$  of 1,250 ft/sec is reached and then the probability values remain the same. Data are not included in figure 11(b) for insertion velocities corresponding to values of  $\Delta V_e$  less than 40 ft/sec. It should be noted that the impact-angle probability distributions for these insertion velocities will fall anywhere between the upper and lower distributions shown in the figure.

#### Translunar Missions

Results of random insertion errors for ballistic trajectories designed to pass the moon at certain distances are shown in figures 12 and 13. In such trajectories with relatively low values of insertion velocity, the vehicle revolves around the far side of the moon and returns to the vicinity of the earth. At insertion velocities above the escape value, the vehicle will pass the moon and enter into an orbit about the sun. Trajectories designed to miss the moon can be considered for controlled lunar landing missions for any value of insertion velocity. The results in figures 12 and 13 apply to all nominal values of insertion radius and insertion flight-path angle.

Perilune-distance probability.— The data in figure 12 include two groups of nominal trajectories: those which are planned to miss the center of the moon by 6,000 statute miles (4,920-statute-mile perilune

altitude) and those planned to miss the center by 1,180 statute miles (100-statute-mile perilune altitude). Results are shown for the three specified sets of standard deviations of insertion errors. Probabilities of hitting the moon are shown in figure 12(a) and probabilities of perilune altitude exceeding the nominal perilune altitude are shown in figure 12(b) as a function of insertion velocity.

The interpretation of the results in figure 12 depends on the definition of a successful mission. When success is defined as not hitting the moon, the data in figure 12(a) indicate that the probability of success is, with one exception, essentially independent of insertion velocity. The one exception applies to trajectories designed for a perilune distance of 1,180 statute miles (100-statute-mile perilune altitude) and having large insertion errors ( $\sigma = 4N$ ). It can also be noted for these low-altitude nominal trajectories with  $\sigma = 4N$  that the probability of hitting the moon is lower (success is greater) than that for trajectories with smaller insertion errors. This lower probability is a result of the fairly large percentage of the trajectories with large errors ( $\sigma = 4N$ ) that pass the moon on the side opposite to that aimed for. The trajectories that pass on this side have counterclockwise motion with respect to the moon and do not return to the earth, but are nevertheless considered successful because they do not strike the moon.

The previously mentioned effect of large insertion error on trajectories designed for low perilune altitudes is reflected in the data of figure 12(b). For this case, the probabilities of exceeding the design (or nominal) perilune altitude (100 statute miles) are considerably above a value of 0.5 because of the fairly large percentage of trajectories that pass the moon at a distance greater than 100 statute miles on the side of the moon opposite to that aimed for. A detailed analysis for this case would show that, for example, at  $\Delta V_e = 1,200$  ft/sec about 50 percent of the trajectories pass the moon at a distance of more than 100 statute miles on the side of the moon aimed for, about 20 percent strike the moon (as shown in fig. 12(a)), a very small percentage come within 100 statute miles of either side of the moon, and the rest (about 30 percent) pass the moon at a distance of more than 100 miles on the side of the moon opposite to that aimed for. For the trajectories in the other cases shown in figure 12(b), the probabilities of exceeding the design altitude are about 0.5 because all or most of these trajectories remain on the side of the moon aimed for.

Perilune-distance dispersion.- When a trajectory, to be considered successful, must pass within a certain distance of the design perilune altitude, the data shown in figure 13 are of interest. In this figure, altitude dispersions about the design (or nominal) perilune altitude are shown plotted against insertion velocity for the three specified sets of insertion errors. The curves are shown for a probability of 0.5. For example, at  $\Delta V_e = 1,200$  ft/sec and  $\sigma = N$ , there is a probability of 0.5 that a trajectory will come within 800 statute miles of the design

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perilune altitude. The data shown for the 100-statute-mile nominal perilune altitude are unrealistic in that most of the dispersion in the direction toward the moon will be within the moon itself. (Dispersions were not determined for this altitude for trajectories with  $\sigma = 4N$ .) The data for the low altitude are included, however, to show that the results of figure 13 will generally apply at any nominal perilune altitude.

The data in figure 13 indicate that, except for very low values of velocity, the altitude dispersion is not affected by the nominal value of insertion velocity. Also, the amount of dispersion is seen to be directly proportional to the magnitude of insertion error.

#### CONCLUDING REMARKS

A statistical study has been made of the effects of random insertion errors on the success of lunar missions. The study included a wide assortment of ballistic trajectories designed for lunar-impact or for lunar-miss missions. Although simple two-body trajectory equations were used, the results are generally applicable to any statistical analysis of lunar-ballistic-trajectory accuracy requirements. The results of the study are summarized as follows:

1. The relative importance of accuracy in each of the insertion conditions is highly dependent on the nominal value of insertion velocity. For trajectories designed for low insertion velocities, accuracy in the velocity is the most important for a successful mission; for high insertion velocities, accuracy in both the lead angle and flight-path angle is important.
2. The nominal value of insertion flight-path angle has essentially no effect on the success of a mission.
3. When the results are normalized on the basis of the minimum insertion velocity required for lunar impact, the nominal value of insertion radius has no effect on the success of a mission.
4. For trajectories designed to hit the moon, the nominal value of insertion velocity has an appreciable effect on the success of a mission. For the relative ratios between the standard deviations of each insertion-condition error used in this analysis, trajectories designed with the insertion velocity below the escape-velocity value afford the best chance of success. An appreciable change in these ratios could change this result.

5. For trajectories designed to miss the moon, the nominal value of insertion velocity generally has no appreciable effect on the success of a mission.

6. For trajectories designed for lunar impact, relatively large dispersions in the longitude of the impact point can be expected for all practical values of insertion accuracies. The nominal values of insertion radius and insertion flight-path angle do not affect impact-longitude dispersion, whereas the value of nominal insertion velocity does.

7. Insertion accuracy has a large effect on the chance of achieving large impact angles. The effects of the nominal values of insertion radius and insertion flight-path angle are negligible and the effect of the nominal value of insertion velocity is small.

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National Aeronautics and Space Administration,  
Langley Air Force Base, Va., February 28, 1962.

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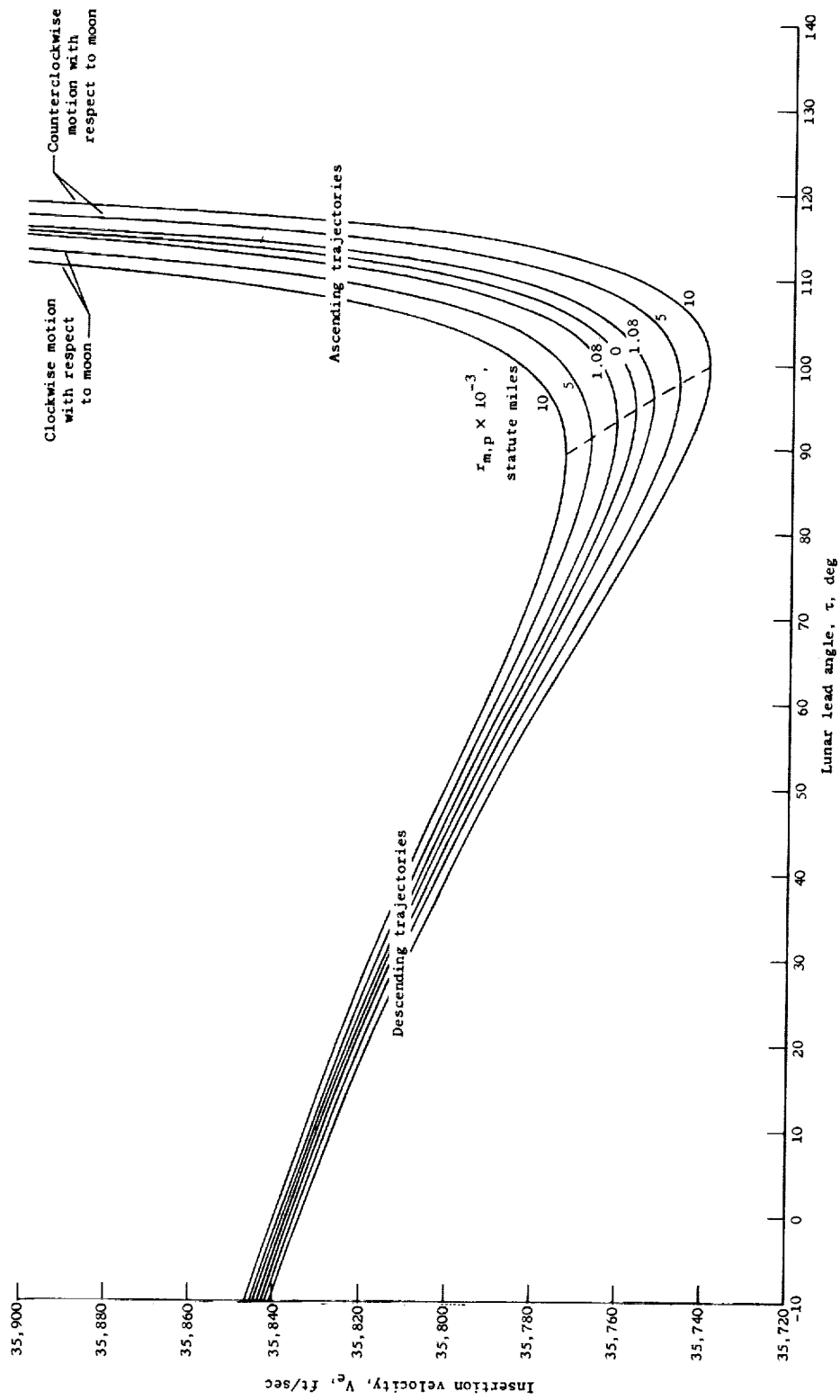
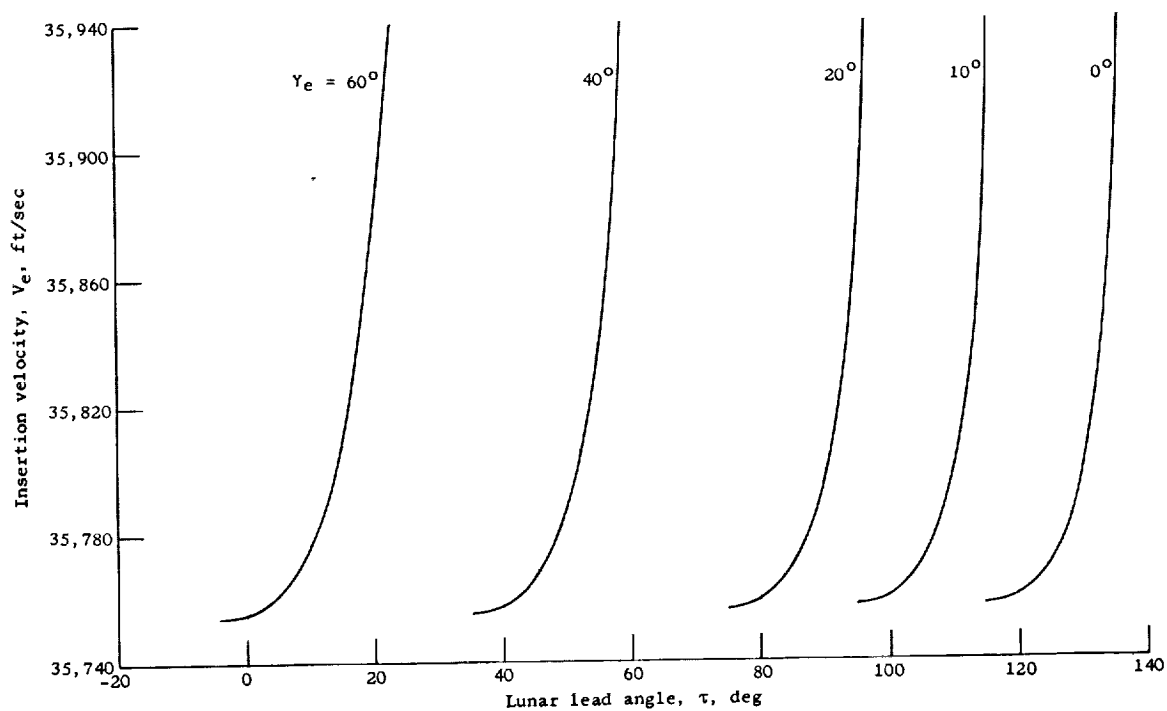
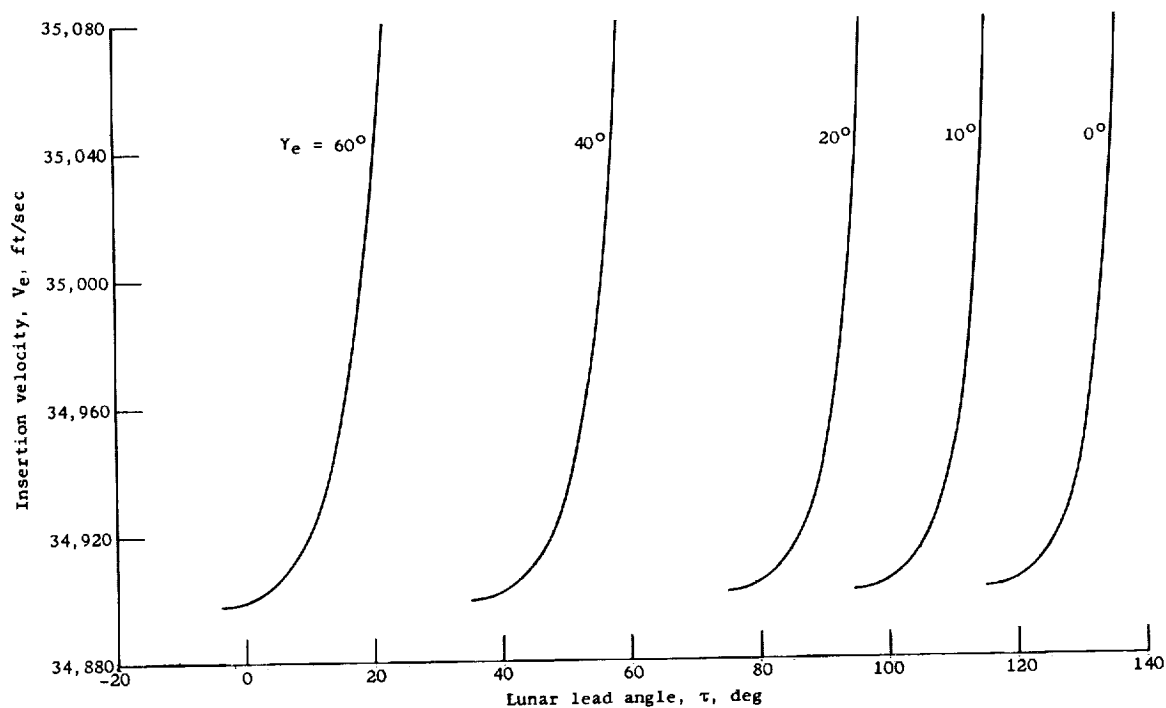


Figure 1.- Example of insertion conditions required for given perilune distances. Insertion radius  $r_e = 4,100$  statute miles; insertion flight-path angle  $\gamma_e = 10^\circ$ . (Direction of motion referenced to looking down on north pole of moon.)



(a) Insertion radius  $r_e = 4,100$  statute miles.



(b) Insertion radius  $r_e = 4,300$  statute miles.

Figure 2.- Examples of insertion conditions required for lunar dead-center hits with ascending trajectories.

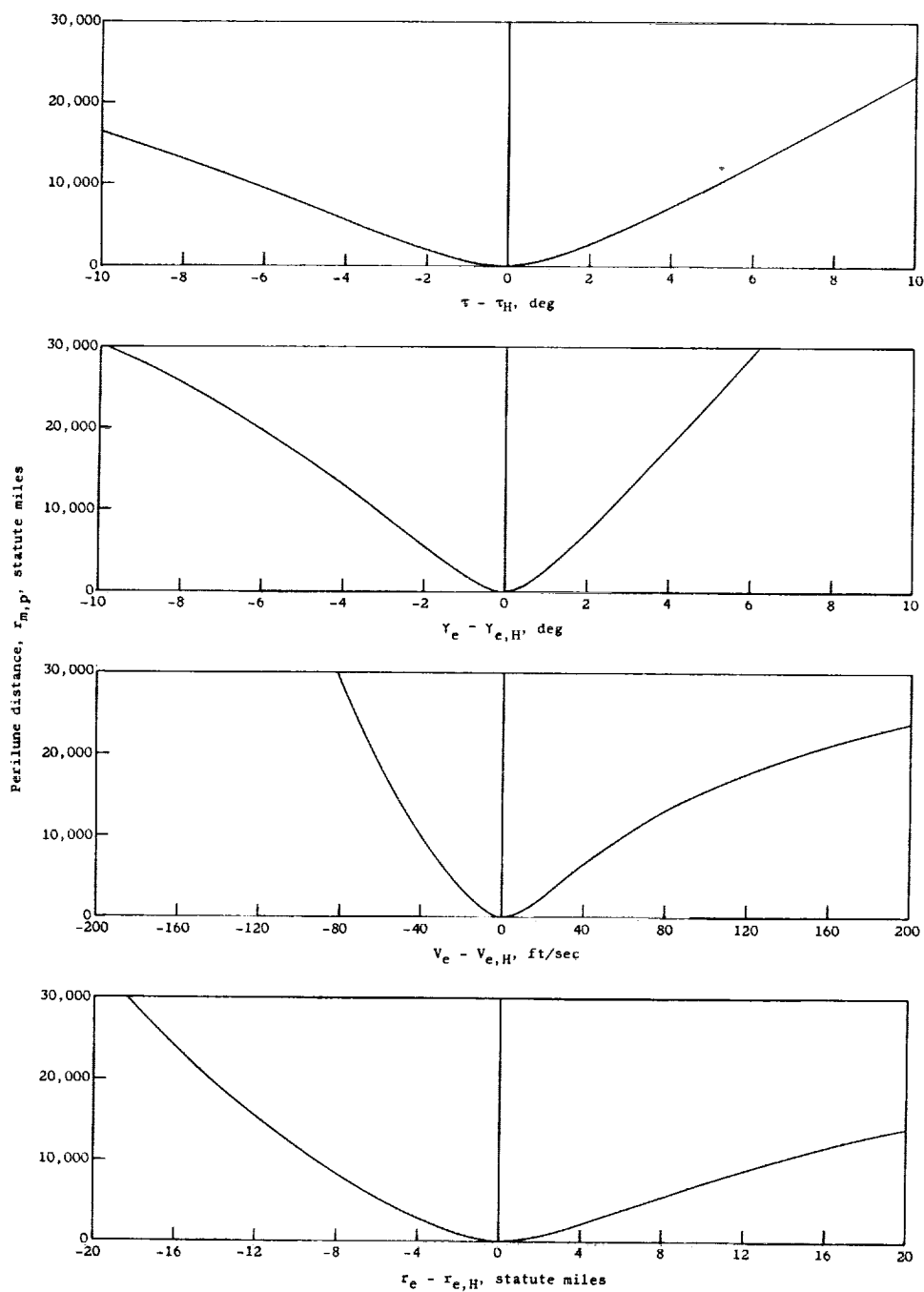
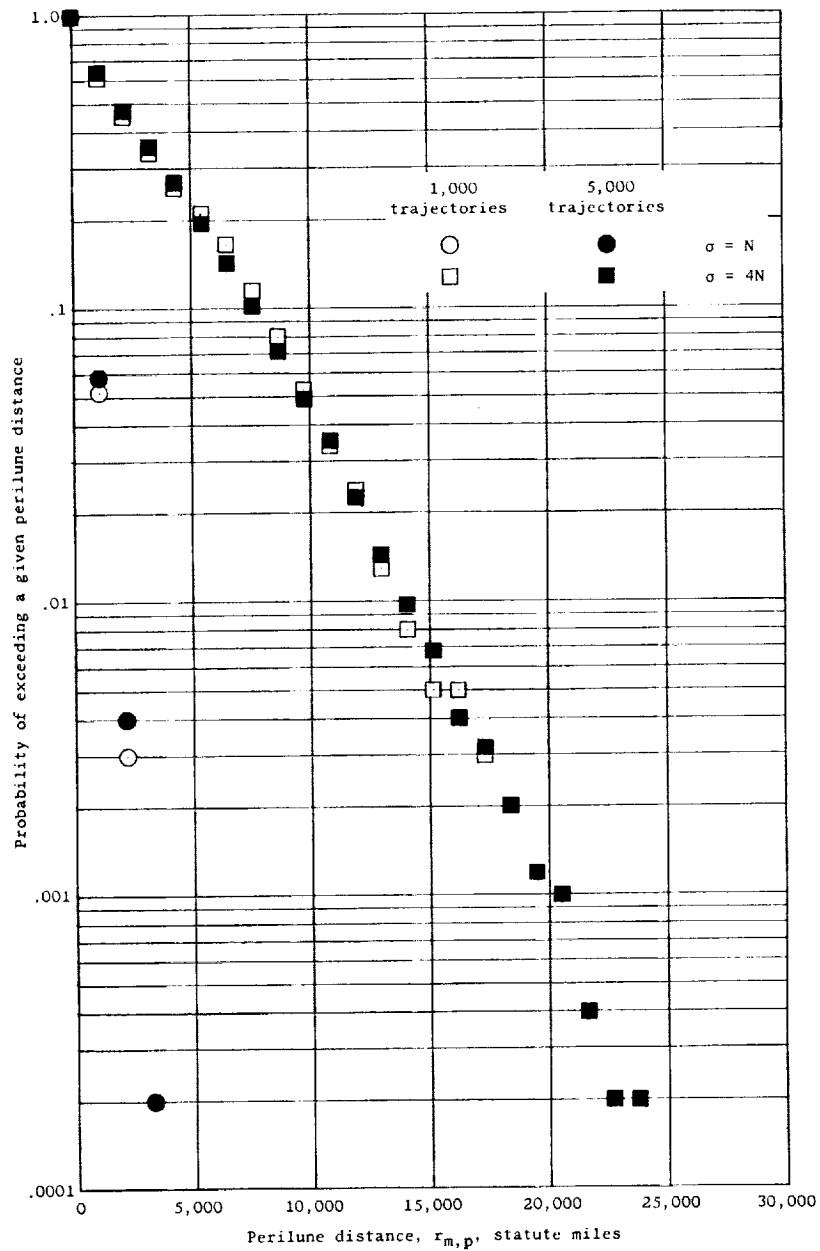


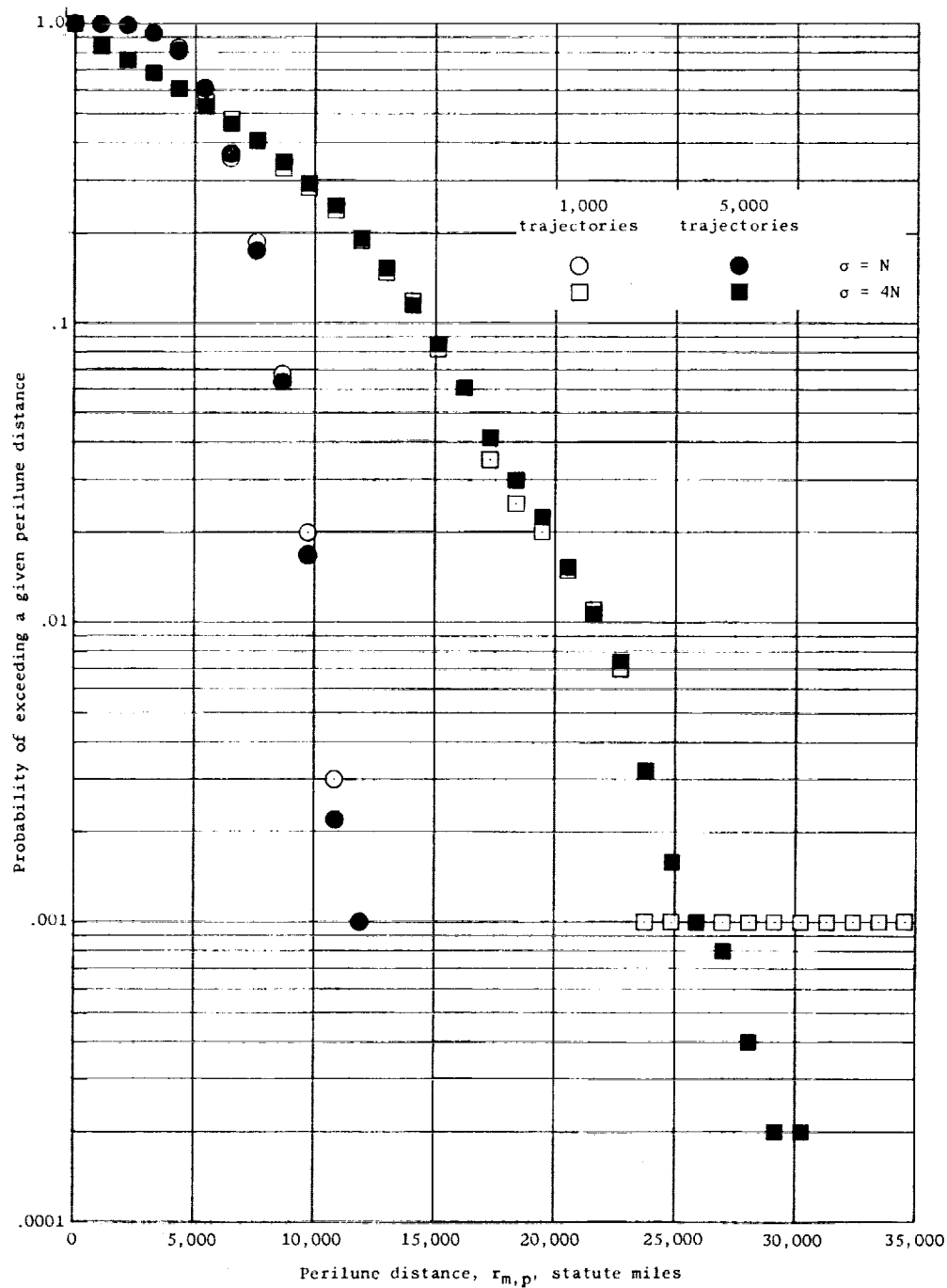
Figure 3.- Example of effects of insertion errors on perilune distance for a trajectory designed to intersect the center of the moon.  
 $\tau_H = 110^\circ$ ;  $\gamma_{e,H} = 10^\circ$ ;  $V_{e,H} = 35,796$  ft/sec;  $r_{e,H} = 4,100$  statute miles.

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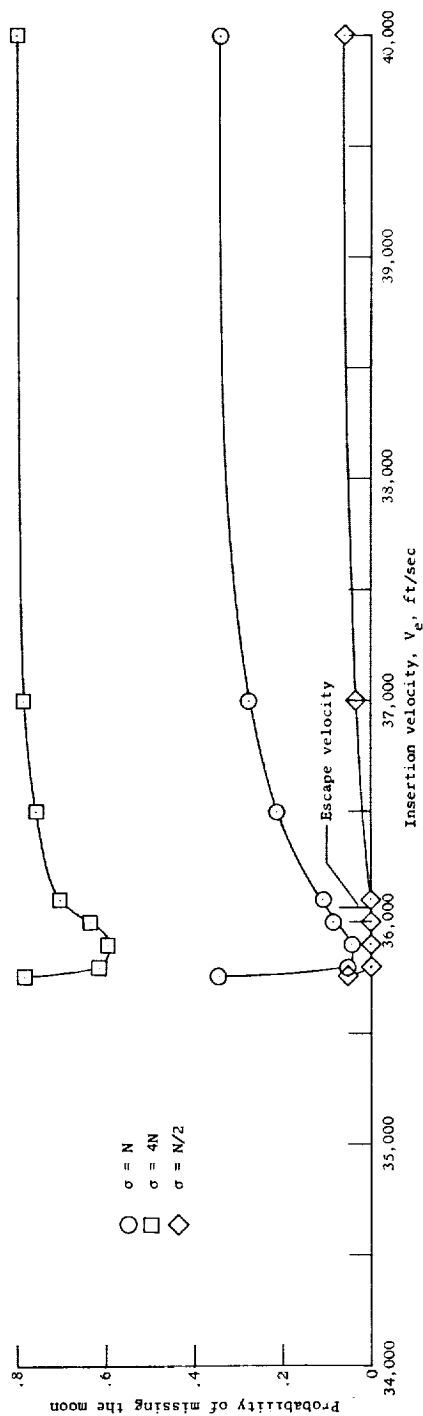
(a) Nominal trajectory results in a dead-center hit  
(Perilune distance = 0).  $\tau = 110^\circ$ .

Figure 4.- Comparison of probability distributions of perilune distance for 1,000 and 5,000 trajectory calculations. Nominal insertion values are:  $r_e = 4,100$  statute miles;  $V_e = 35,796$  ft/sec; and  $\gamma_e = 10^\circ$ .

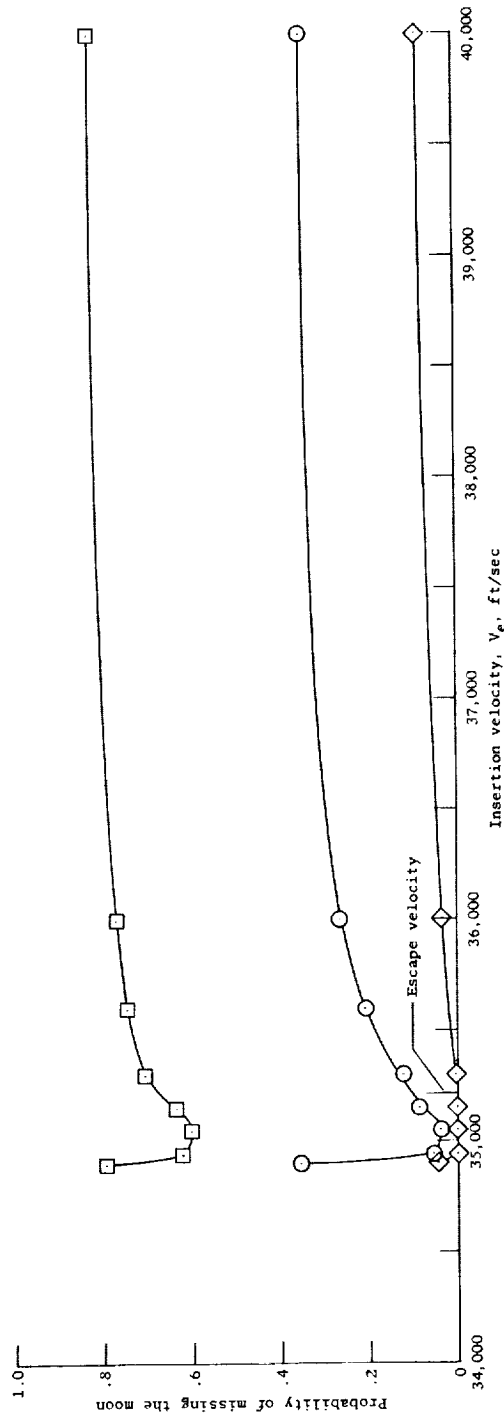


(b) Nominal trajectory results in a perilune distance of 6,000 statute miles (perilune altitude of 4,920 statute miles).  $\tau = 105.83^\circ$ .

Figure 4.- Concluded.



(a) Insertion radius  $r_e = 4,100$  statute miles.



(b) Insertion radius  $r_e = 4,300$  statute miles.

Figure 5.- Effect of insertion velocity on probability of missing the moon for various insertion radii and standard-deviation magnitudes of insertion error. Nominal trajectory results in a dead-center hit.  $\gamma_e = 10^\circ$ .

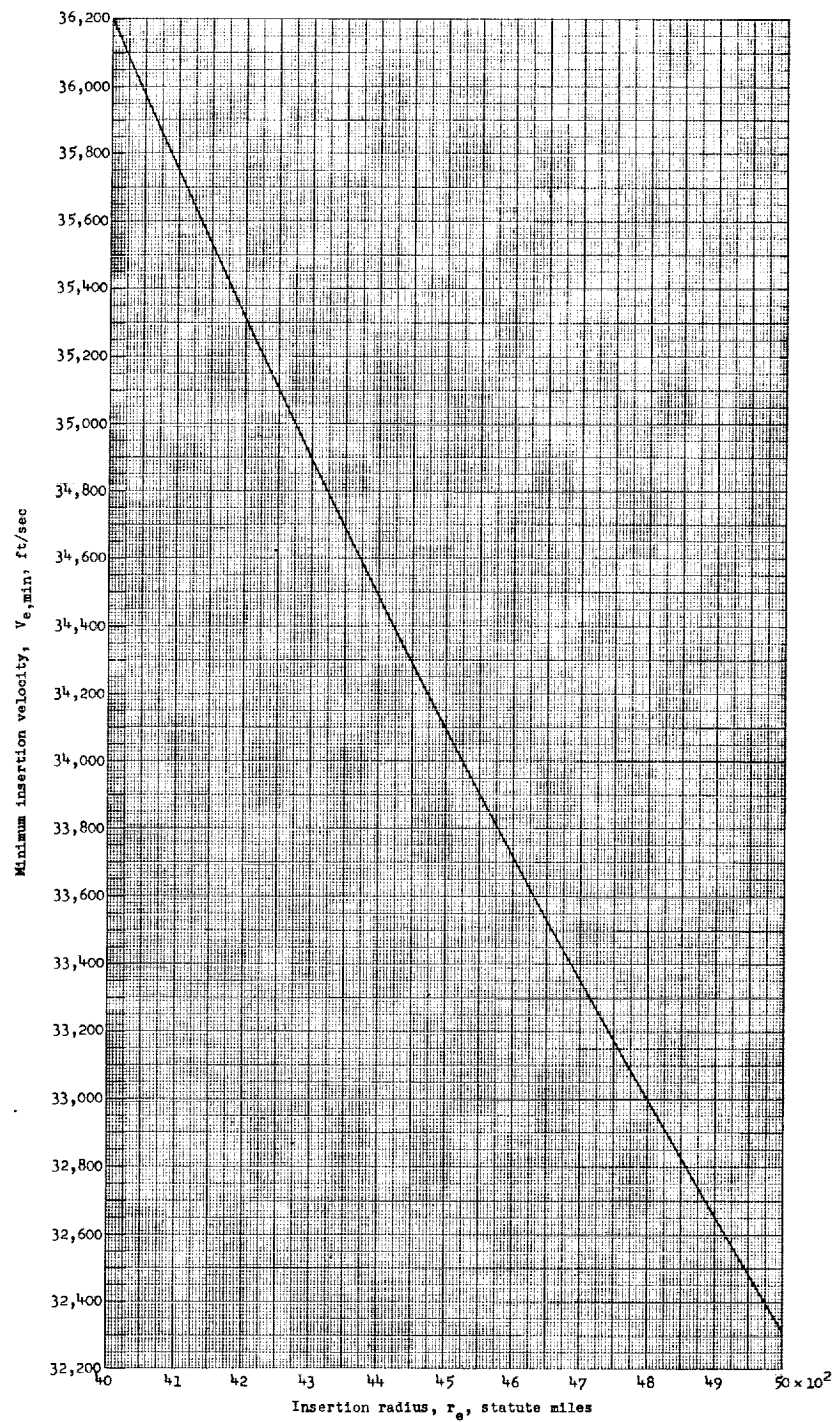


Figure 6.- Minimum insertion velocity for lunar impact. Insertion flight-path angle  $\gamma_e = 0^\circ$  to  $60^\circ$ .

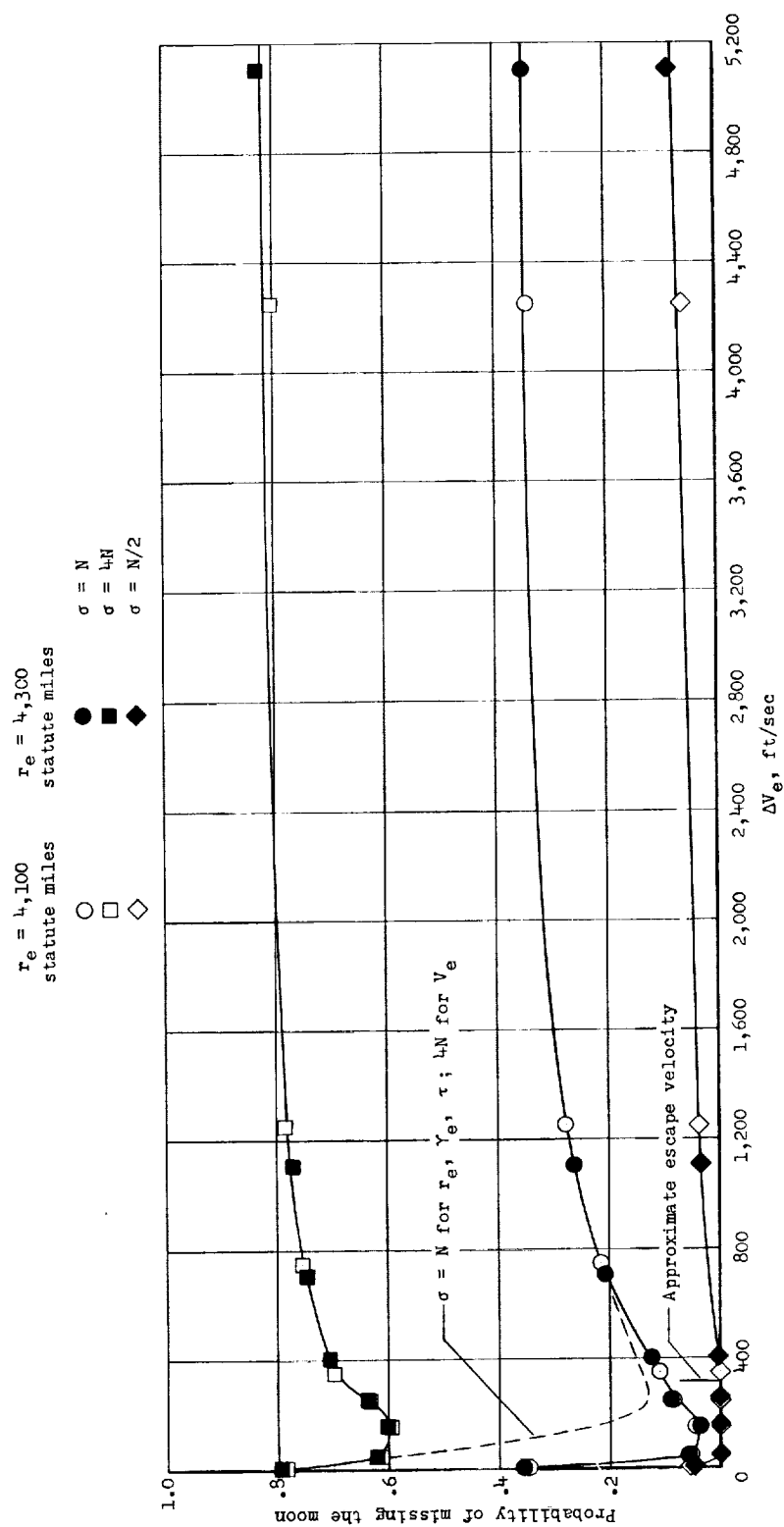


Figure 7.- Effect of insertion velocity on probability of missing the moon for various standard-deviation magnitudes of insertion error. Nominal trajectory results in a dead-center hit.  $\gamma_e = 100^\circ$ .

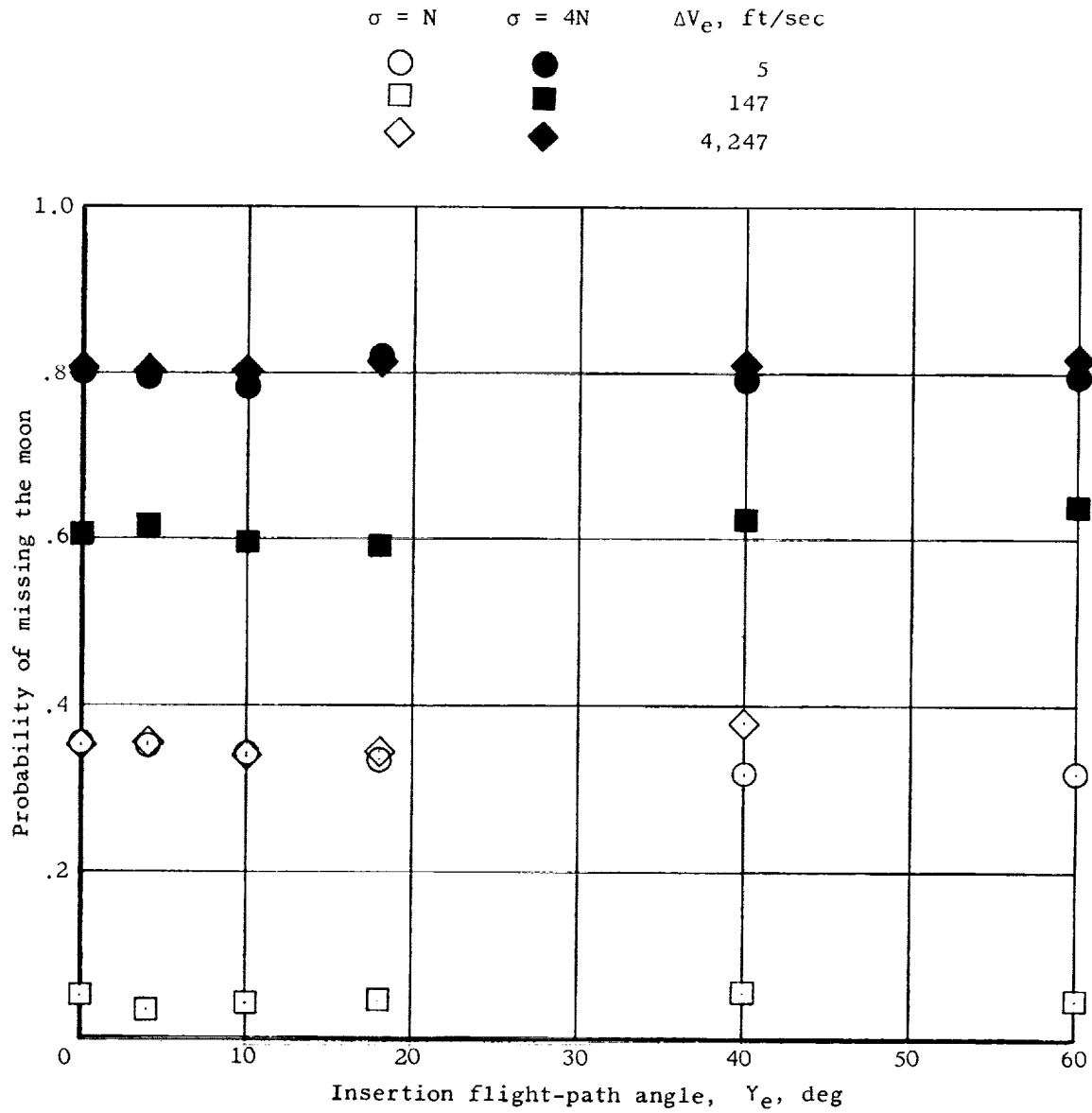


Figure 8.- Effect of insertion flight-path angle on probability of missing the moon for various standard-deviation magnitudes of insertion error. Nominal trajectory results in a dead-center hit.

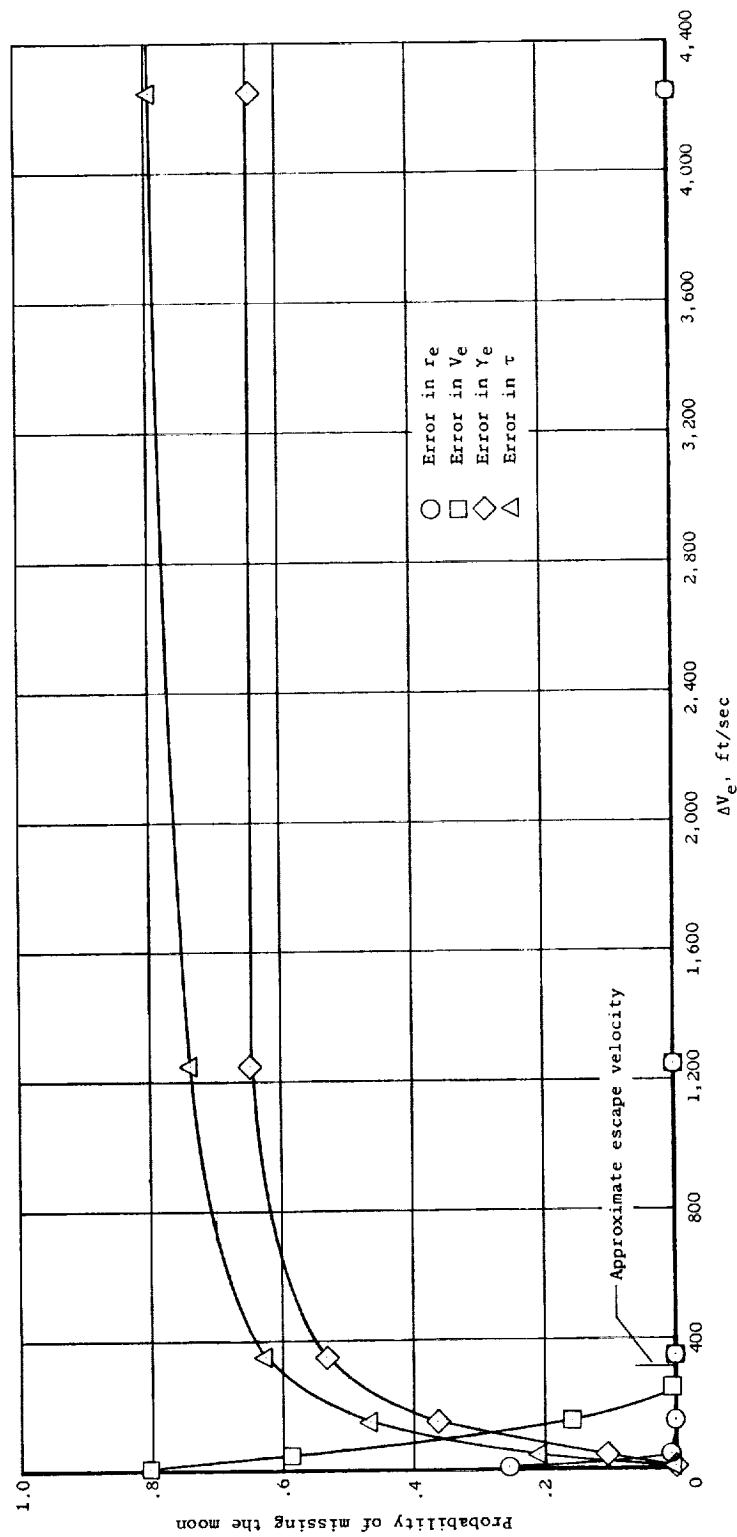


Figure 9.- Effects of random errors in a single insertion condition on probability of missing the moon as a function of insertion velocity.  $\sigma = 4N$ . Nominal trajectory results in a dead-center hit.

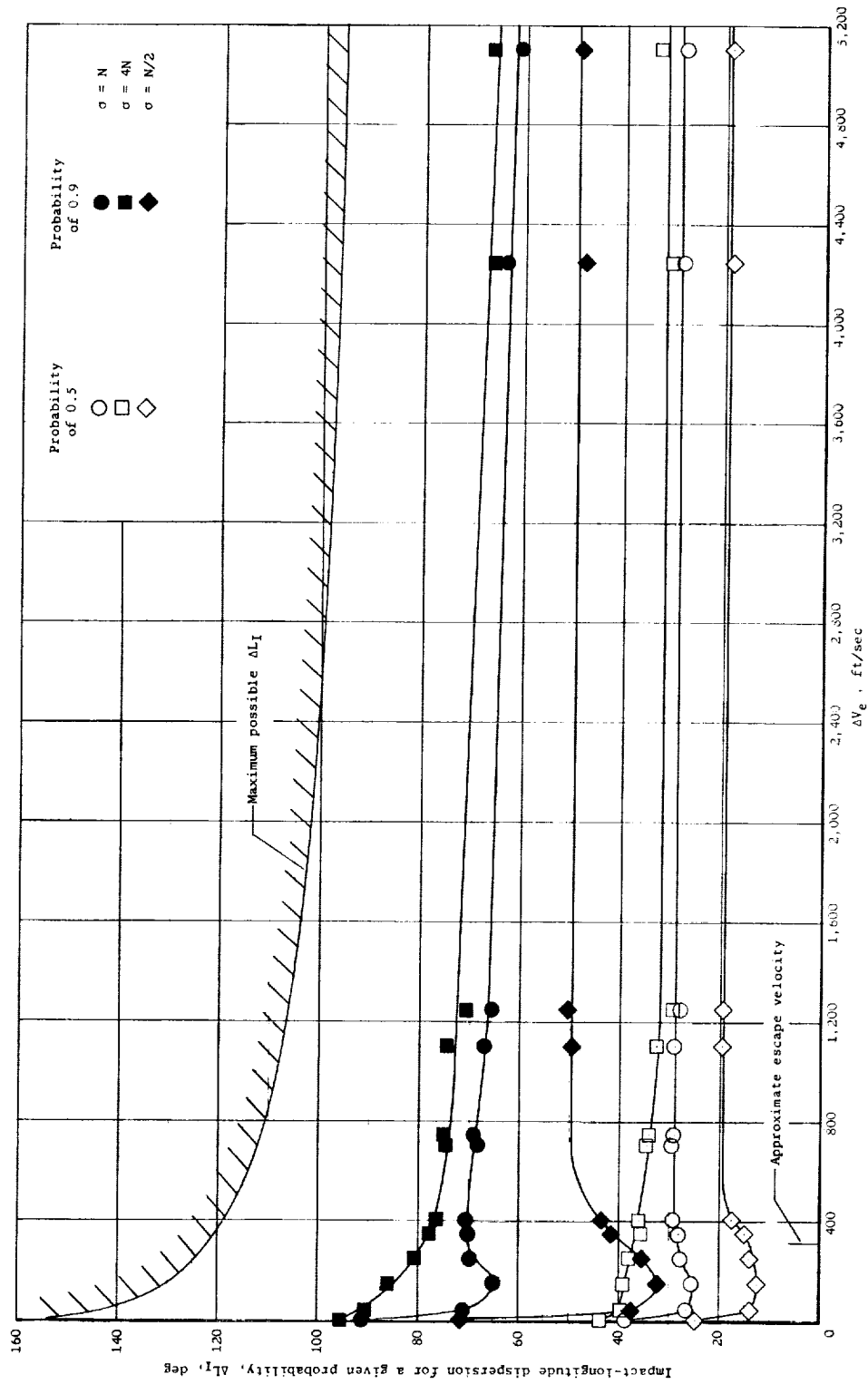
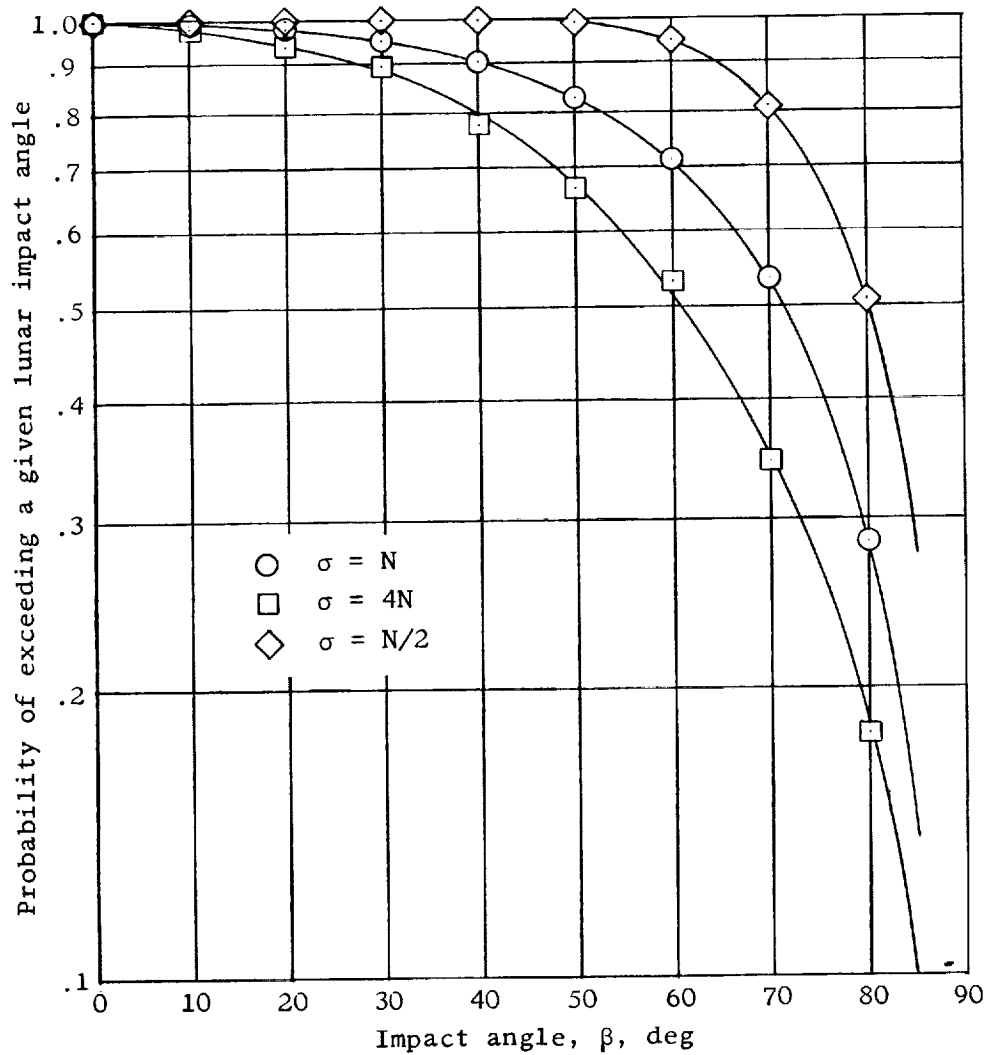
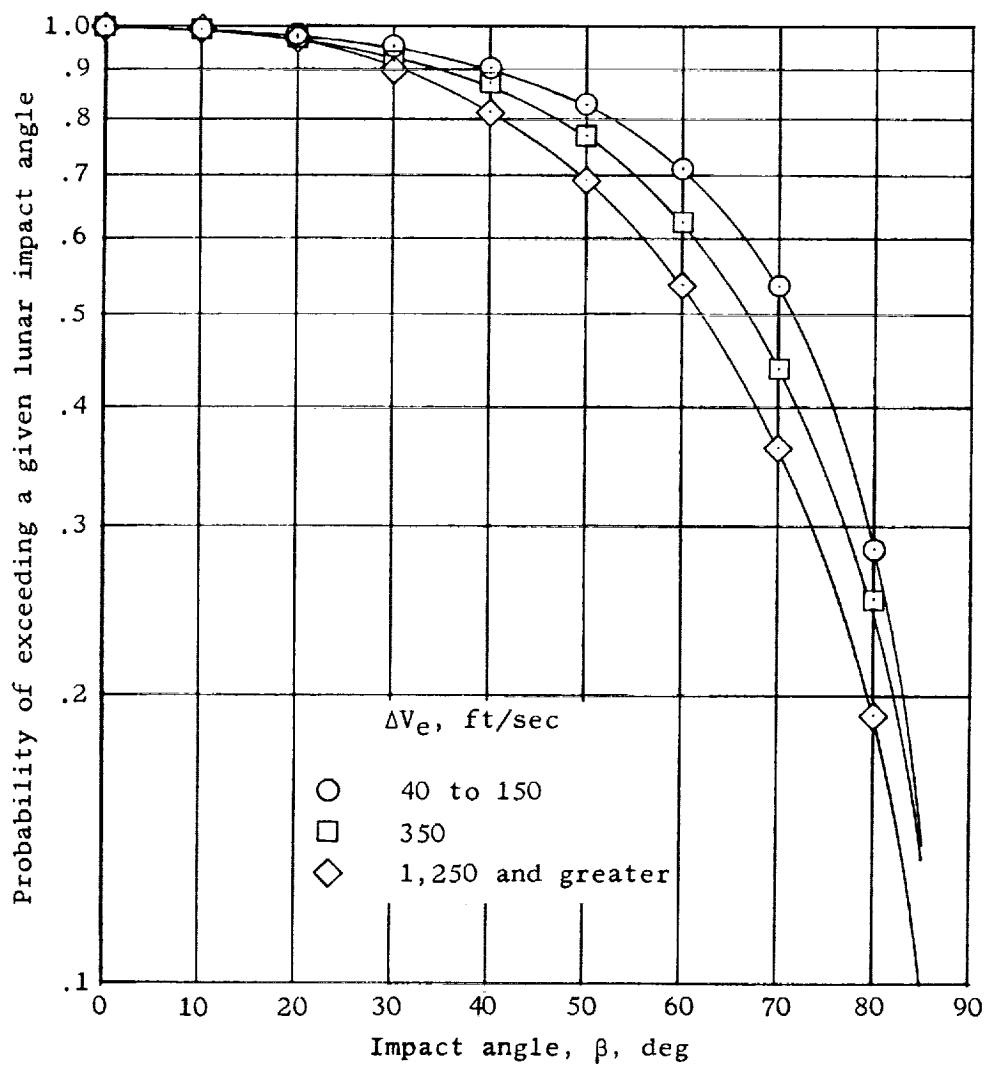


Figure 10.- Effect of insertion velocity on dispersion in lunar-impact longitude for various standard-deviation magnitudes of insertion error. Nominal trajectory results in a dead-center hit.



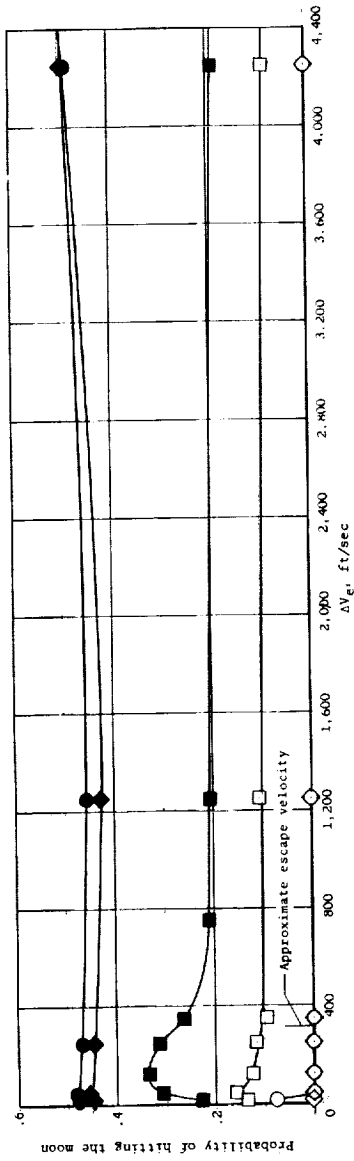
(a) Effect of standard deviation of insertion error.  $\Delta V_e = 150$  ft/sec.

Figure 11.- Probability distributions of lunar impact angle. Nominal trajectory results in a dead-center hit (Impact angle =  $90^\circ$ ).

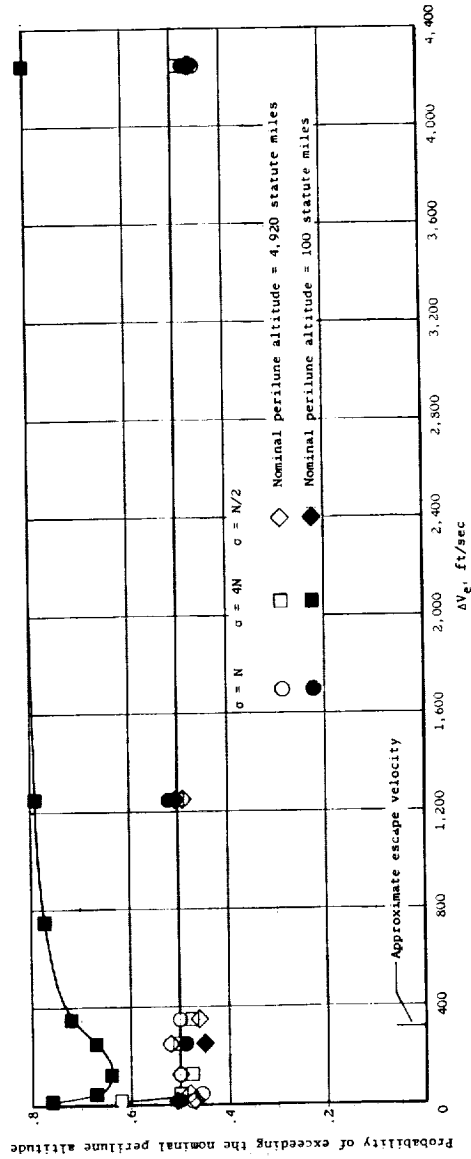


(b) Effect of insertion velocity.  $\sigma = N$ .

Figure 11.- Concluded.



(a) Probability of hitting the moon for trajectories designed to miss the moon.



(b) Probability that nominal perilune altitude will be exceeded.

Figure 12.- Effect of insertion velocity on perilune distance for various standard-deviation magnitudes of insertion error. Nominal trajectories result in perilune distances of 6,000 and 1,180 statute miles (perilune altitudes of 4,920 and 100 statute miles).

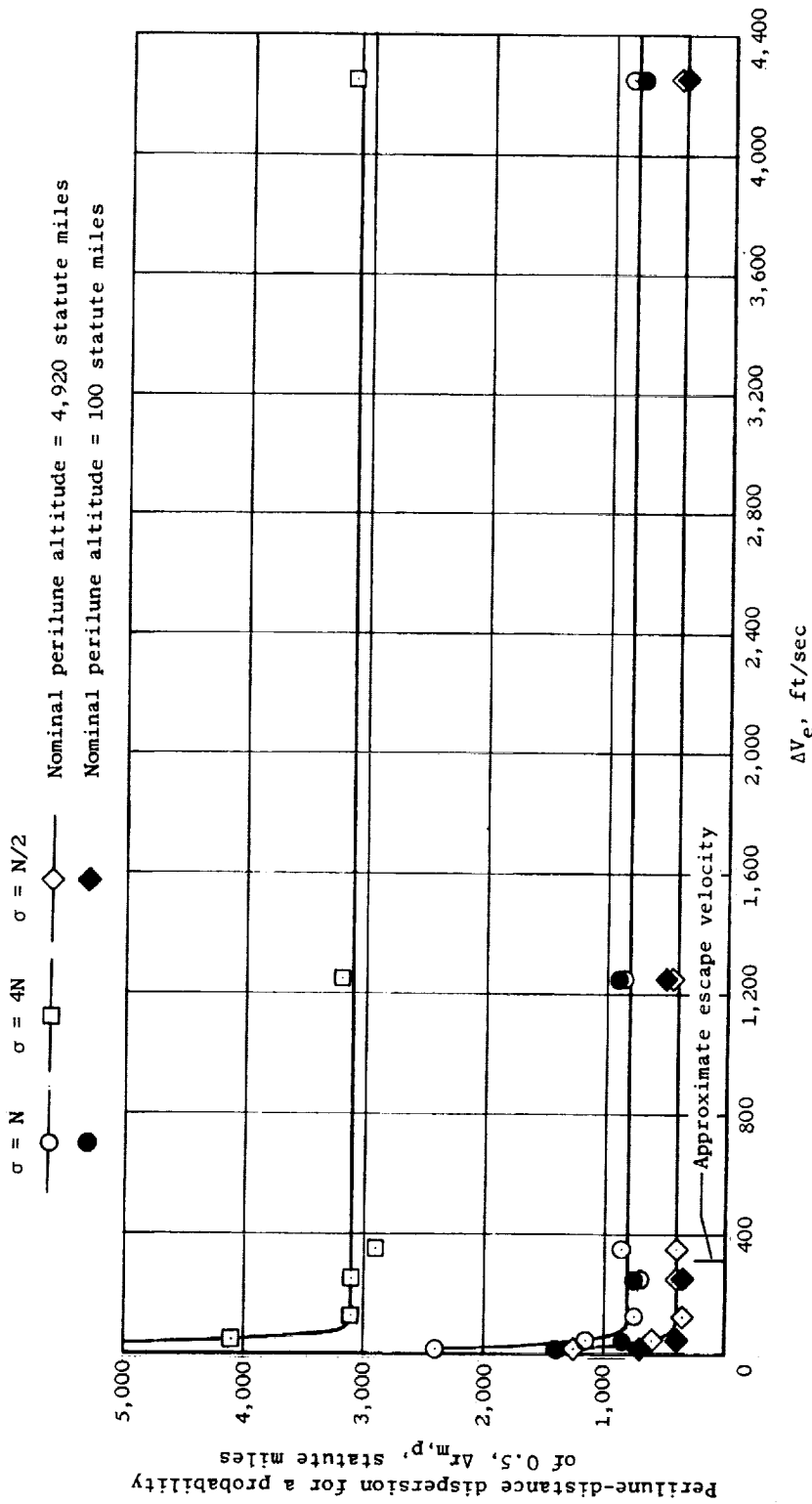


Figure 13.- Effect of insertion velocity on dispersion in perilune distance for various standard-deviation magnitudes of insertion error. Nominal trajectories result in perilune distances of 6,000 and 1,180 statute miles (perilune altitudes of 4,920 and 100 statute miles).

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